A CAUSAL PERSPECTIVE ON JUMP-DIFFUSION FOR TIME-SERIES ANOMALY DETECTION

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ABSTRACT

Time series anomaly detection is essential for maintaining robustness in dynamic real-world systems. However, most existing methods rely on static distribution assumptions, while overlooking the latent causal structures and structural shifts that underlie real-world temporal dynamics. This often leads to poor explanation of anomalies and misclassification of environment-induced variations. To address these shortcomings, we propose Causal Soft Jump Diffusion Anomaly Detection (CSJD-AD), a novel framework that models both latent dynamics and soft-gated expected jumps through a structural jump diffusion process. We adopt a causal perspective grounded in environment-conditioned invariance by inferring discrete environment states and conditioning the jump-augmented process on them, yielding a practical detector for unlabeled sensor streams without aiming to identify true interventions. By generating paired counterfactual and factual trajectories, the model explicitly contrasts causally consistent behavior with unexplained deviations. Our method achieves state-of-the-art performance across benchmark datasets, demonstrating the importance of incorporating causal reasoning and jump-aware dynamics into time series anomaly detection.

1 INTRODUCTION

Time series anomaly detection (TSAD) plays a pivotal role in modern data analysis by identifying unexpected or irregular patterns within sequential data streams. In industry, it enables predictive maintenance by spotting abnormal sensor readings, while in finance, it helps detect fraud through unusual trading behaviors Yang et al. (2024); Livernoche et al. (2023). Beyond these domains, anomaly detection is also indispensable in applications such as quality control, e-commerce analytics, environmental monitoring of smart grids, and Internet of Things infrastructure Pinaya et al. (2022); Yang et al. (2024). As time series data grows in volume and complexity, robust and adaptive detection methods become indispensable. Machine learning and deep models have demonstrated enhanced accuracy and scalability over traditional statistical techniques, navigating challenges such as seasonality, noise, and evolving patterns. Thus, developing advanced, robust, and context-aware anomaly detection models is both timely and essential for maintaining reliability and enabling proactive decision-making in real-world systems Blázquez-García et al. (2021).

Recent advances in deep learning have greatly enhanced TSAD by harnessing neural networks' expressive power. Recurrent models Bontemps et al. (2016); Ergen & Kozat (2019) are widely used to capture temporal dependencies and forecast future values, with deviations from predicted trajectories serving as anomaly indicators. Convolutional Neural Networks (CNNs) Ren et al. (2019); Yang et al. (2023)are also employed to extract local temporal patterns and Transformer Song et al. (2018); Yue et al. (2022) have shown strong performance in long-range sequence modeling, enabling better detection in datasets with complex seasonal and contextual dependencies. Generative approaches, including GAN-based detectors Du et al. (2021); Zhou et al. (2019), have been applied to learn the distribution of normal sequences, using discriminator feedback or likelihood-based scoring to detect anomalies.

Despite these success, most methods assume a stationary data-generating process and overlook latent causal structures, even though real-world environments often exhibit distribution shifts driven

by discrete changes in underlying causal mechanisms Carvalho et al. (2023). Unlike images or event logs with curated labels, most operational time series are raw sensor outputs without environment/state annotations. The regime that determines what is normal is latent, piecewise-constant, and changes at unknown times, so identical observations can be benign in one regime and faulty in another. For example, in industrial monitoring, a spike in machine temperature may be expected during active operational load but highly abnormal during scheduled maintenance or idle states, despite having similar marginal statistics. These context-dependent variations are not anomalies by itself, but reflect different underlying environment regimes. Absent an explicit regime model, detectors routinely misclassify benign shifts as anomalies, degrading alert reliability and obscuring why alarms fire. Reconstruction- and density-based methods are especially vulnerable because they score deviations from a stationary reference rather than regime-conditioned normality.

We address this gap with an environment-conditioned jump-diffusion model that introduces a discrete environment variable E to encode the regime governing observation generation dynamics conditional on E under a causally motivated invariance perspective, making drift and volatility stable within each regime and flagging only structural violations. Unlike traditional jump diffusion process Merton (1976), our soft-gated jumps model real-world anomalies more effectively. We replace the fixed exogenous jump process by parameterizing the jump activation probability as $p_E = f_\psi(U, E)$, where f_ψ conditions on the latent state and the inferred environment. Rather than sampling a binary gate, we inject the expected jump contribution $p_E\, J_E(U_t, E_t)$ directly at each macro-step boundary. This design learns context sensitive jump timing from data, keeps activations sparse by suppressing jumps in stable regimes and increasing them under regime driven volatility, and improves anomaly discrimination by separating structural changes from environment induced variation.

Building upon this environment-aware jump diffusion formulation, we introduce a mechanism for modeling the causal invariance objective through dual latent trajectory generation. In our framework, using only the drift and diffusion terms conditioned on the current environment E, we simulate how the system would evolve if no abrupt perturbation occurred. The counterfactual trajectory, constructed using only these components, models how the system evolves under its current environment regime. As such, it already accounts for all expected or structured changes that arise as part of normal transitions across operating conditions. In contrast, the factual trajectory introduces a jump term that is deterministically weighted via a learned gating propensity. These jumps represent infrequent, irregular deviations that cannot be explained by the environment-driven dynamics alone.

By explicitly separating causally consistent transitions from unexplained deviations, our model, CSJD-AD defines a principled training signal: the discrepancy between factual and counterfactual trajectories quantifies structural violations. This causal contrastive loss focuses learning on environment-invariant irregularities, enhancing the model's sensitivity to meaningful anomalies. In short, the main contributions of this papers are summarized as follows:

- We introduce a discrete environment variable E that, under an invariance perspective, conditions the drift, diffusion, and gated expected jump to disentangle environment-consistent changes from true anomalies.
- We propose an environment-conditioned jump diffusion formulation with a learnable soft gating mechanism that jointly models smooth dynamics and abrupt structural transitions, enabling context-aware and interpretable anomaly detection.
- We construct dual latent trajectories, factual and counterfactual, to improve shift–robust anomaly detection via conditional invariance.
- A unified training objective integrates reconstruction fidelity, variational stability, and causal contrast, resulting in a robust and interpretable framework for TSAD.

2 Related Work

Pattern-Deviation Methods: This family of methods detect anomalies by measuring how much a subsequence deviates from learned global or local patterns. They define normality based on statistical regularities, neighborhood structures, or clustering, and flag subsequences as anomalous if they exhibit low likelihood, weak pattern similarity, or sparse local density. For example NormA Boniol et al. (2021), which computes anomaly scores based on the weighted distance of time series subsequences to clustered normal patterns, and Series2Graph Boniol & Palpanas (2020), which

constructs a transition graph of subsequence patterns and detects anomalies via low-degree and low-weight graph trajectories.

Forecasting-based Methods: These methods Ding et al. (2018); Dai & Chen (2022) detect anomalies by learning to predict future points or subsequences from recent observations. A prediction model is trained on normal data, and anomalies are identified when actual observations deviate significantly from their predicted values. Typically, a sliding window is used to forecast one point at a time, making this approach well-suited for streaming settings where anomalous events are rare. For example, AD-LTI Wu et al. (2020) detects anomalies by combining seasonal decomposition with GRU forecasting and introduces a Local Trend Inconsistency score to account for unreliable historical trends. DeepAnt Munir et al. (2018) is a lightweight CNN-based model that detects point and contextual anomalies with minimal training data and tolerates mild data contamination. GTA Chen et al. (2021) uses transformers and graph convolutions to model temporal and inter-sensor dependencies in multivariate time series for semi-supervised anomaly detection.

Reconstruction-based Methods: These methods detect anomalies by learning to reconstruct normal time series patterns. Unlike forecasting models, they use full context, including the current input, for richer representations. Trained on normal subsequences via sliding windows and latent embeddings, they flag anomalies by identifying high reconstruction errors or low reconstruction probabilities during inference, capturing subtle deviations from expected behavior. For example, VAE-GAN Niu et al. (2020) combines variational autoencoding and adversarial learning to detect anomalies using both reconstruction errors and discriminator feedback in a semi-supervised setting. TranAD Tuli et al. (2022) enhances transformer-based anomaly detection with adversarial training to amplify subtle anomalies and uses self-conditioning to improve stability and generalization.

3 METHODOLOGY

3.1 PROBLEM SETTING

We address TSAD under both semi-supervised and unsupervised paradigms. In the semi-supervised setting, the model is trained on normal data to learn a representation of typical temporal dynamics, then identifies deviations caused by faults or external disruptions as anomalies during inference. Formally, given an observed series $X = \{x_1, \ldots, x_T\}$ with $x_t \in \mathbb{R}^d$ ($d = 1$ for univariate and $d > 1$ for multivariate cases), the objective is to capture the structure of normal behavior and detect departures. We also evaluate our approach in an unsupervised setting, where anomalies are detected solely based on the intrinsic properties of the data without reliance on labeled normal samples.

3.2 VARIATIONAL CAUSAL ENCODER

Given an observed time series segment $X \in \mathbb{R}^{T\times d}$, our goal is to encode it into two types of latent representations: a latent mapping matrix $U \in \mathbb{R}^{T\times k}$ capturing the temporal data's underlying dynamics, each row U_t summarizes step-t latent features and each column indexes a latent channel shared across the window, and a discrete environment variable $E \in \Delta^{K-1}$ (a probability simplex over K environments) that serves as an unsupervised index for regime-conditioned latent dynamics and conveys causal semantics through conditioning and counterfactual-style simulation rather than structural identification.

We achieve this via a shared neural encoder CausalEncoder(X) that produces the variational posteriors: $q_\phi(U \mid X), \quad q_\phi(E \mid X)$, where ϕ denotes the encoder parameters. Specifically, U is sampled from a Gaussian distribution with learnable mean and variance:

$$q_\phi(U \mid X) = \mathcal{N}(\mu_U(X), \mathrm{diag}(\sigma_U^2(X))), \tag{1}$$

and E is sampled using the Gumbel-Softmax reparameterization to approximate a categorical distribution in a differentiable manner, yielding a soft one-hot vector that is then used to to condition downstream network components:

$$q_\phi(E \mid X) = \mathrm{GumbelSoftmax}(\mathrm{logits}_E(X)), \tag{2}$$

each regime k therefore parameterizes its own drift, diffusion, and jump functions; we use the subscript $(\cdot)_E$ to denote conditioning on E, conveying causal semantics via environment-conditioned invariances.

3.3 CAUSAL SOFT JUMP DIFFUSION SDE

Building on our motivation, the need to separate causally consistent regime shifts from true anomalies, we now present the formal dynamics that drive our latent representations. Throughout, we retain the intuition of jump diffusion processes while adopting fully differentiable formulation.

3.3.1 FROM JUMP DIFFUSION TO SOFT JUMP DIFFUSION

In the classical jump diffusion framework Merton (1976), the latent state $U_t \in \mathbb{R}^d$ evolves according to

$$dU_t = \mu(U_t)\,dt \;+\; \sigma(U_t)\,dW_t \;+\; J(U_t)\,dN_t, \tag{3}$$

where N_t is a Poisson process of fixed rate and each jump contributes a discrete increment of size $J_E(U_t)$. However, the discrete sampling to Poisson variables breaks gradient flow, complicating end-to-end learning. Neural Jump SDEs Jia & Benson (2019) and the NJDTPP Zhang et al. (2024) are built for event prediction, they tie jumps to observed event and train by maximizing event-time likelihood, which limits their use on densely sampled sensor streams without event timestamps.

To reconcile expressive power with differentiability and utilize a mechanism to decide dynamically when a jump should or should not occur based on the current context, we model the latent state $U_t \in \mathbb{R}^d$ as a continuous diffusion process interspersed with instantaneous soft jumps at macro-grid times $\{\tau_j\}_{j=0}^{J}$. Concretely, for $j = 0, \ldots, J-1$ we write

$$\begin{cases} dU_t \;=\; \underbrace{\mu_E\big(U_t, E\big)}_{\text{drift net}}\,dt \;+\; \underbrace{\sigma_E\big(U_t, E\big)}_{\text{diffusion net}}\,dW_t, \\ U_{\tau_{j+1}} \;=\; U_{\tau_{j+1}^-} \;+\; \underbrace{p_E\big(U_{\tau_{j+1}^-}, E\big)}_{\text{soft gate}}\;\underbrace{J_E\big(U_{\tau_{j+1}^-}, E\big)}_{\text{jump net}}, \end{cases} \tag{4}$$

where $U_{\tau_{j+1}^-} \;=\; \lim_{t\uparrow\tau_{j+1}} U_t$.

For each $t \in (\tau_j, \tau_{j+1}]$, the latent state follows a diffusion driven by a Brownian motion W_t. At $t = \tau_{j+1}$, we apply an instantaneous soft jump of magnitude $p_E(U_{\tau_{j+1}^-}, E)\, J_E(U_{\tau_{j+1}^-}, E)$. In practice, we restrict to one expected soft jump per window. This aligns the model with the windowing granularity used for evaluation and serves as a first-moment approximation of the cumulative jump effect of a compound-Poisson process over the window, $\int J\,dN \;\approx\; p_E J_E$. Here the scalar gate $p_E \in (0,1)$ encodes the propensity of a structural shock conditioned on the current latent state and environment E_{t_k}, whereas J_E specifies its direction and scale. Positive entries are excitatory and negative entries are inhibitory. Detailed statements and proofs are provided in Appendix B.

3.4 CAUSAL PATH GENERATION

We evolve the window-level matrix state $U_s \in \mathbb{R}^{T\times k}$ along solver time $s \in [0,1]$ while preserving its $T \times k$ layout. Given the encoded latent state U_0 and environment E, we evolve the process over M micro-steps of size $\delta t = \Delta t/M$:

$$U^{(m+1)} \;=\; U^{(m)} + \mu_E\big(U^{(m)}, E\big)\,\delta t + \sigma_E\big(U^{(m)}, E\big)\sqrt{\delta t}\,\epsilon_m, \quad \epsilon_m \sim \mathcal{N}(0, I), \tag{5}$$

with $U^{(0)} = U_0$. The last state $U^{(M)}$ is the counterfactual latent, denoted U_{CF}.

Then we inject the expected jump contribution,

$$U_{\mathrm{F}} \;=\; U_{\mathrm{CF}} + p_E\big(U_{\mathrm{CF}}, E\big)\; J_E\big(U_{\mathrm{CF}}, E\big), \tag{6}$$

where $p_E \in (0,1)$ is a learnable propensity and J_E encodes jump magnitude. This implements one expected soft jump at the window boundary per window, consistent with the macro grid in the section 3.3.

Finally, we blend the two trajectories $U_{\mathrm{final}} = U_{\mathrm{CF}} + \gamma(U_{\mathrm{F}} - U_{CF}),\ \gamma \in [0,1]$, and reconstruct the observation via $X_{\mathrm{gen}} = \mathrm{Decoder}(U_{\mathrm{final}})$, where γ controls the strength of jump influence, allowing smooth interpolation between counterfactual and factual paths. We define γ as a hyperparameter and the settings are provided in Table 8 of Appendix D.

This soft-jump formulation preserves the causal intuition of jump diffusion, $p_E J_E$ increases only in volatile regimes while remaining fully differentiable.

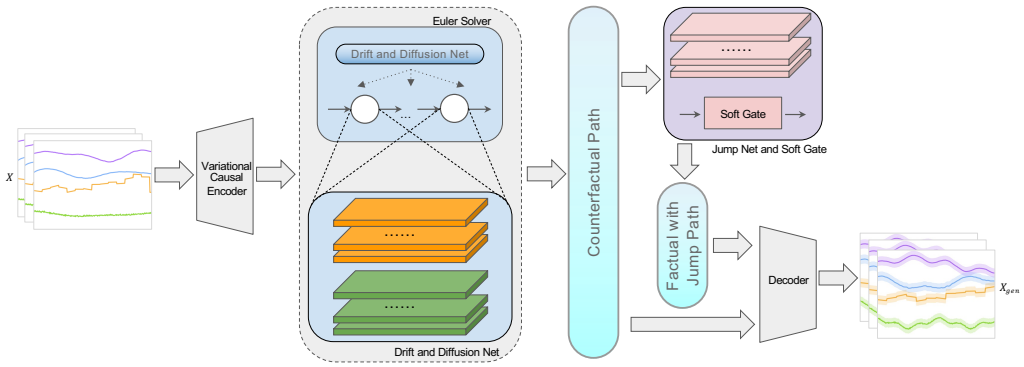

Figure 1: Overall model structure. X is encoded into latent U and environment E, evolved by drift–diffusion dynamics to a counterfactual path, perturbed by a gated jump to yield the factual path, blended into U_{final}, and finally decoded back to X_{gen}.

3.5 Inference Training with Counterfactual Loss

Building on the dual-path simulation framework, we introduce a principled loss formulation that enforces meaningful causal representations and stable variational inference.

Causal Discrepancy Weight. Before introducing the losses, we define a causal discrepancy weight that modulates the contrastive term during training. It quantifies the magnitude of the environment-conditioned jump and its improbability. The weight is

$$\mathcal{W}_{\text{CD}}(X) = \|J_E\|_1 \cdot (1 - p_E). \tag{7}$$

Loss Components. We minimize the total loss:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{recon}} + \lambda_{\text{causal}} \cdot \mathcal{L}_{\text{causal}} + \lambda_{\text{kl}} \cdot (\mathcal{L}^U_{\text{KL}} + \mathcal{L}^E_{\text{unif}}), \tag{8}$$

where λ_{causal} and λ_{kl} control regularization strength.

The reconstruction loss $\mathcal{L}_{\text{recon}} = |X - X_{\text{gen}}|^2$, ensures that latent variables capture observable patterns, enabling anomaly detection via reconstruction error.

The causal contrastive loss $\mathcal{L}_{\text{causal}} = |U_{\text{F}} - U_{\text{CF}}|^2 \cdot \mathcal{W}_{\text{CD}}(X)$, promotes consistency between factual and counterfactual latent paths in stable regimes, while allowing divergence when jumps occur, regularizing behavior and supporting unsupervised anomaly scoring.

Finally, the KL terms Kingma & Welling (2013); Pereyra et al. (2017) regularize the latent space and the environment variable:

$$\mathcal{L}^U_{\text{KL}} := D_{\text{KL}}(q_\phi(U|X)\|\mathcal{N}(0, I)), \quad \mathcal{L}^E_{\text{unif}} := \mathbb{E}_{q_\phi(E|X)}\left[\sum_{e=1}^{K} E_e \log\big(E_e + \varepsilon\big)\right], \tag{9}$$

here $\mathcal{L}^E_{\text{unif}}$ is a negative-entropy regularizer on $q_\phi(E \mid X)$, promoting smooth latent representations. Together, these losses ensure stable training and improve generalization across diverse regimes.

3.6 CSJD-AD Overall Pipeline

Figure 1 shows the full CSJD-AD pipeline. The variational encoder maps each input window X to a latent state U and a environment code E. A drift network $\mu_E(\cdot)$ and diffusion network $\sigma_E(\cdot)$, both conditioned on E, advance U through an Euler–Maruyama step to generate the counterfactual trajectory U_{CF}. A jump module, likewise conditioned on E, outputs a jump magnitude $J_E(U_{\text{CF},E})$ and gate $p_E(U_{\text{CF},E})$; adding the gated jump yields the factual state U_{F} as described in Equation(6). The model blends the two paths via a coefficient γ to obtain the final latent U_{final}, which the decoder transforms back into X_{gen}. Training minimizes the reconstruction error, the causal contrastive loss, and KL regularization on both U and E.

Table 1: Statistics of the seven anomaly-detection datasets. AR stands for Anomaly Ratio.

Dataset	Dimension	Entity	Train	Test	AR
ASD	19	12	37,089	49,452	0.295
ECG	2	9	6,999	2,851	0.153
MSL	55	27	58,317	73,729	0.198
SMD	38	28	878,560	702,848	0.131
WADI	127	1	784,173	172,604	0.073
Yahoo	1	56	30,456	7,614	0.036
KPI	1	26	396,211	566,316	0.095

At inference, we use the total objective as an energy score, $\mathcal{S}(X) = \mathcal{L}_{\text{total}}(X)$, treating the KL-type terms as data-dependent posterior complexity penalties; constants (e.g., $\log K$) are dropped and $\lambda_{\text{causal}}, \lambda_{\text{kl}}$ are fixed from training. For completeness, we report the variant in Table 7 in Appendix D.

4 Experiment

4.1 Experiment Setup

Benchmark Datasets. We evaluate our model on five multivariate time series anomaly detection datasets: ASD Li et al. (2021), ECG Keogh et al. (2005), MSL Hundman et al. (2018), SMD Su et al. (2019a), and WADI Ahmed et al. (2017), and two univariate datasets: Yahoo Laptev et al. (2015) and KPI Li et al. (2022), all with point-wise anomaly labels. Table 1 illustrates the details for each dataset. The multivariate datasets follow a semi-supervised setting, assuming access to anomaly-free training data. In contrast, the univariate datasets lack predefined train/test splits, requiring manual partitioning. As a result, we cannot guarantee the absence of anomalies in the training sets, placing these datasets in an unsupervised setting. For Yahoo and KPI, we exclude entities that contain no anomalies in the test set, since the F1 score would otherwise be undefined.

Evaluation Metrics. We evaluate each model using the standard F1 score and the average Area Under the Precision-Recall Curve (AUCPR) across entities. We do not use point-adjusted F1 because it credits an entire anomaly segment when any single point crosses the threshold, which can push random or diffuse predictions to high F1 and inflate scores on long segments Kim et al. (2022).

Many datasets contain multiple entities without aligned timestamps, so we train models separately for each. Since F1 is not additive, unlike many baselines that average per-entity F1, we aggregate true/false positives and negatives across entities and recompute the F1 from the combined confusion matrix. For multi-entity datasets, we report the mean and standard deviation of AUCPR. For WADI, which has only one entity, AUCPR standard deviation is not available.

Since CSJD-AD integrates a jump-diffusion SDE with explicit Euler rollout, which raises natural concerns about stepwise simulation cost, we report runtime and GPU memory to demonstrate feasibility, and we observe mid-range efficiency comparable to recent 2024 and 2025 deep learning baselines. The details are provided in Table 6 in Appendix A.3.

Baseline Models. We evaluate eleven TSAD methods, including VAE-based models (LSTM-VAE Park et al. (2018), OmniAnomaly Su et al. (2019b)), transformer-based approaches (TranAD Tuli et al. (2022), PUAD Li et al. (2023), AnomalyTran Lai et al. (2023a), NPSR Lai et al. (2023b), Dual-TF Nam et al. (2024), Sensitive-HUE Feng et al. (2024)), a diffusion-based model D^3R Wang et al. (2023), a hybrid diffusion–TCN model IGCL Zhao et al. (2025) and a CNN-MLP model RedLamp Obata et al. (2025).

Table 2: Time-series anomaly-detection performance on seven public benchmarks (higher is better). Best scores are in **bold**; second–best are underlined. The Table 10 in Appendix include Precision and Recall as an extended version.

Method	Metric	Multivariate Benchmarks					Univariate Benchmarks	
		ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
LSTM-VAE	F1	0.327	0.274	0.407	0.367	0.248	0.326	0.182
	AUCPR	0.245±.180	0.206±.150	0.285±.249	0.395±.257	0.139	0.255±.152	0.135±.120
OmniAnomaly	F1	0.238	0.216	0.271	0.459	0.229	0.340	0.201
	AUCPR	0.175±.132	0.154±.152	0.149±.182	0.365±.202	0.120	0.245±.218	0.140±.010
AnomalyTran	F1	0.425	0.464	0.344	0.304	0.102	0.372	0.303
	AUCPR	0.281±.201	0.306±.221	0.236±.237	0.273±.232	0.040	0.261±.182	0.204±.139
TranAD	F1	0.305	0.461	0.420	0.386	0.263	0.484	0.287
	AUCPR	0.238±.178	0.368±.251	0.278±.239	0.412±.260	0.139	0.691±.324	0.285±.206
D^3R	F1	0.253	0.301	0.197	0.326	0.117	0.201	0.152
	AUCPR	0.150±.110	0.180±.131	0.138±.101	0.228±.167	0.070	0.120±.080	0.090±.061
PUAD	F1	0.351	0.382	0.384	0.364	0.259	0.301	0.284
	AUCPR	0.280±.203	0.304±.221	0.307±.102	0.291±.210	0.155	0.240±.172	0.224±.152
NPSR	F1	0.350	0.451	0.373	0.372	0.613	0.550	0.321
	AUCPR	0.281±.200	0.405±.281	0.336±.241	0.335±.245	0.552	0.495±.344	0.288±.160
Dual-TF	F1	0.661	0.538	0.127	0.287	0.551	0.352	0.126
	AUCPR	0.628±.212	0.511±.182	0.124±.126	0.215±.074	0.523	0.317±.190	0.124±.126
Sensitive-HUE	F1	0.366	0.309	0.451	0.397	0.699	0.281	0.170
	AUCPR	0.340±.188	0.410±.245	0.432±.121	0.462±.283	0.641	0.489±.429	0.227±.253
IGCL	F1	0.022	0.094	0.223	0.208	0.014	0.201	0.208
	AUCPR	0.079±.066	0.183±.141	0.179±.102	0.126±.132	0.218	0.300±.277	0.206±.198
RedLamp	F1	0.205	0.165	0.284	0.113	0.624	0.299	0.057
	AUCPR	0.154±.103	0.200±.196	0.199±.290	0.128±.140	0.564	0.653±.409	0.089±.129
Ours	F1	**0.676**	**0.586**	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**
	AUCPR	**0.682±.193**	**0.627±.161**	**0.464±.296**	**0.637±.183**	**0.653**	**0.937±.200**	**0.342±.242**

4.2 Overall Experiment Results

Table 2 reports F1 and AUCPR. The extended table with precision and recall, together with training settings and resource usage, appears in Appendices A and D. We fix the window size to 200 on all datasets to limit hyperparameter effects. While many baselines tune the window per dataset, our model delivers strong and consistent results without such tuning.

Our model achieves state-of-the-art performance across all time series anomaly detection benchmarks, consistently outperforming existing methods in both F1 and AUCPR metrics. As shown in Table 2, our model demonstrates strong robustness under severe class imbalance. For instance, on the Yahoo and ECG datasets—both characterized by extremely low anomaly ratios—we achieve AUCPR scores of 0.937 and 0.627, respectively. These represent relative improvements of over 20% compared to the next-best models (TranAD with 0.691 on Yahoo and Dual-TF with 0.511 on ECG), highlighting the model's superior ability to maintain precision and recall in imbalanced settings.

Beyond multivariate benchmarks, our method performs strongly on univariate datasets, demonstrating adaptability across data regimes and flexibility in semi-supervised and unsupervised settings over a broad range of temporal dimensionalities.

4.3 Ablation Study

4.3.1 Componets Effectiveness

We evaluate four ablated variants of our model by disabling each key component in isolation, while keeping all other settings fixed. The w/o U_{F} variant removes the factual path U_{F} and omits the causal loss $\mathcal{L}_{\text{causal}}$ accordingly. The w/o p_E variant removes the gating head and injects a deterministic jump once per window. The w/o E variant replaces the causal encoder with a standard encoder which only outputs the continuous latent variable U, and disabling the KL loss $\mathcal{L}_{\mathrm{KL}}$. In the w/o $\mathcal{L}_{\text{causal}}$, we

Table 3: Ablation study of the proposed model. Each column reports F1 scores (higher is better). **Bold** numbers denote the best result for that dataset. The Table 11 in Appendix is the extended version that includes the AUCPR results.

Variant	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
w/o U_{F}	0.562	0.573	0.463	0.563	0.682	0.662	0.194
w/o p_E	0.675	**0.604**	0.445	0.568	0.696	0.698	0.310
w/o E	0.571	0.552	0.455	**0.575**	0.678	0.618	0.118
w/o $\mathcal{L}_{\text{causal}}$	**0.682**	0.566	0.464	0.563	0.685	0.703	0.250
w/o $\mathcal{L}_{\mathrm{KL}}$	0.680	0.578	0.465	0.571	0.681	0.655	0.188
Default	0.676	0.586	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**

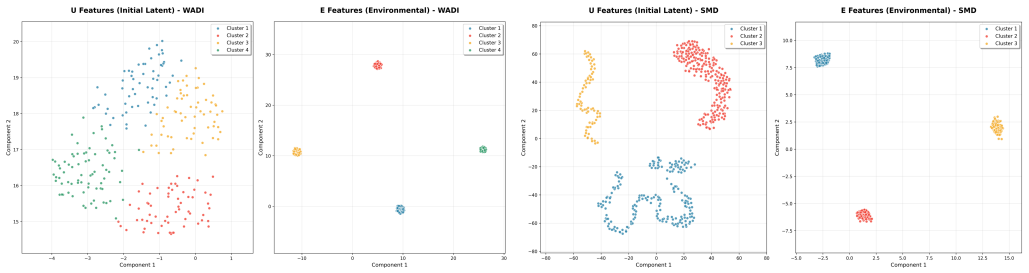

Figure 2: The two UMAP plots on the left show the embedded latents and environmental latents from the Causal Encoder for the WADI dataset. The two UMAP on the right present the corresponding latents for the SMD dataset.

retain the computation of U_{F} and U_{CF} but omit the trajectory contrastive learning objective during training. Finally, the w/o $\mathcal{L}_{\mathrm{KL}}$ variant disables both the Gaussian-prior KL penalty and the entropy regularization, while preserving the causal encoder that extracts the environmental variable E.

As shown in Table 3, most ablations lead to a consistent decline in detection performance across all seven datasets. Notably, some ablated variants still achieve performance comparable to existing state-of-the-art methods on certain benchmarks. For instance, on the SMD dataset, the w/o E variant performs similarly to the full model, suggesting that SMD may contain a single dominant environmental regime, thereby diminishing the benefit of explicit environment modeling. Additionally, on the ASD dataset, removing $\mathcal{L}_{\text{causal}}$ and $\mathcal{L}_{\mathrm{KL}}$ constrain yields slightly better performance; however, the environmental variable E, the factual path U_{F}, and the counterfactual path U_{CF} remain essential components, as their presence continues to support overall model performance.

4.3.2 Causal Environment Representation Quality

We test whether the encoder discovers discrete regimes by projecting the learned embeddings with UMAP and clustering with K-Means using the preset E. We run K-Means with K equal to the pre-specified environments and color each point by its cluster assignment. As Figure 2 shows, the plots of E form K tight, well-separated clouds, confirming that the model has encoded each environment into a distinct region of the latent simplex. By contrast, the U embeddings for WADI appear as scattered but well-separated clouds, whereas SMD forms coherent arcs. In both cases, coloring each U point by $\arg\max(E)$ shows that every latent falls strictly within its corresponding E cluster. This confirms that it always respects the discrete regimes encoded by E even when U is diffuse or varying. Overall, these results validate that (1) our encoder disentangles a small number of causal regimes in E, and (2) the primary latent U varies within each regime, exactly as designed.

To testify that the model uses the environment code E at inference rather than treating it as a redundant head. We train the model normally with learned E. At test time we compare three settings: Default uses each window's inferred E; Single-E forces all windows to the most frequent E; Shuffled-E randomly permutes the inferred E across windows and recomputes the score. As Table 4

Table 4: Effect of modifying learned environment settings on F1 performance (higher is better). Best scores for each dataset within a block are shown in **bold**. Table 12 in Appendix is an extended version that includes AURCPR. Default corresponds to training with correctly learned environment variables.

Strategy	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
Single E	0.654	0.585	0.463	**0.575**	0.693	**0.703**	0.306
Shuffled E	0.539	0.470	0.404	0.396	0.685	0.501	0.143
Default	**0.676**	**0.586**	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**

Table 5: Effect of noise and missing-value settings on F1 performance (higher is better). Best scores for each dataset within a block are shown in **bold**. Default corresponds to training with no added noise or missing values. The Table 13 in Appendix is the extended version that includes the AUCPR results.

Strategy	Level	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
Noise level	0.10	0.652	0.584	0.451	0.569	0.697	0.669	0.218
	0.05	0.642	0.594	0.458	0.574	**0.723**	0.694	0.277
	0.01	**0.703**	**0.600**	0.457	0.575	0.715	**0.707**	**0.353**
Missing ratio	0.40	0.631	0.476	0.454	0.602	0.544	0.702	0.343
	0.20	0.605	0.476	0.455	0.599	0.656	0.702	0.343
	0.10	0.602	0.477	0.453	**0.605**	0.662	0.702	0.343
Default	—	0.676	0.586	**0.467**	0.575	0.701	0.703	0.346

shows, default gives the best F1 on all datasets, Shuffled-E lowers F1 by about 0.13 on average, and Single-E is closer but still worse by 0.025 on average. These results show that correct environment assignments matter and that collapsing or misassigning E degrades performance.

4.4 Robustness under Data Perturbations

We evaluated the robustness of our model against two common data corruptions: additive **Gaussian noise** ($N(0, \sigma), \sigma \in 0.1, 0.05, 0.01$) and random **missing values** ($r \in 40\%, 20\%, 10\%$, imputed by mean). As Table 5 shows, mild noise often improved performance (e.g., ECG 0.586→0.594/0.600, WADI 0.701→0.723/0.715), with only minor drops at $\sigma = 0.1$ and scores still above baselines. Similarly, SMD benefited from masking (0.575→0.605/0.599/0.602), while other datasets showed <10-point losses yet remained superior to competitors. These results highlight the framework's robustness under realistic perturbations.

5 Conclusion and Future Work

In conclusion, we propose a causal structural jump-diffusion framework that unites continuous latent modeling with discrete environments, yielding state-of-the-art anomaly detection and enhanced interpretability. By pairing counterfactual and factual trajectories, our model, CSJD-AD quantifies regime, specific impacts and adapts to structural shifts via a jump-augmented SDE. This causal separation explains why an alarm is raised and delivers state-of-the-art detection performance across seven benchmarks. The approach thus offers a new, interpretable direction for TSAD by explicitly linking latent dynamics to regime-specific causal structure.

In future work, we aim to boost adaptability in regime-agnostic settings by introducing a nonparametric prior, allowing the posterior to automatically shrink unused regimes and infer the number of environments K directly from data, thereby improving robustness when true regimes are unknown. We will also pursue cross-environment generalization by using leave-one-environment-out training with invariant or distributionally robust objectives and by enabling light test-time adaptation of E.

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A Appendix

A.1 Anonymous source code

Code is available at `https://anonymous.4open.science/r/CSJD-AD`.

A.2 Training Resources

All experiments were carried out on a single desktop workstation with the following hardware and software configuration:

- **Operating System**: Ubuntu 24.04 LTS
- **CPU**: AMD Ryzen 9 9950X3D
- **System Memory**: 64 GB DDR5
- **GPU**: NVIDIA GeForce RTX 4090 (24 GB VRAM)
- **Libaries**: Python 3.8 + Pytorch 2.4.1 + CUDA 12.1

The training time and resources usage is listed in Table 6.

A.3 Training Settings

To ensure consistency across experiments and to minimize the impact of individual hyper-parameter choices, we *fixed* the sliding-window length to 200 for *all* datasets—even though their respective optimal windows differ (details in Appendix section 5). Each model was trained for up to 200 epochs with early stopping, allowing adaptive convergence on each dataset. Anomaly thresholds were selected via a grid search that maximised the F1 score, thereby reducing sensitivity to threshold choice.

Table 7 lists the hyper-parameters shared by every experiment; the remaining dataset-specific settings are given in Table 8.

A.4 Datasets Sources

ASD	`https://github.com/zhhlee/InterFusion/tree/main/data`
ECG	`https://www.cs.ucr.edu/~eamonn/discords/ECG_data.zip`
MSL	`https://www.kaggle.com/datasets/patrickfleith/nasa-anomaly-detection-dataset-smap-msl`
SMD	`https://github.com/NetManAIOps/OmniAnomaly/tree/master/`
WADI	`https://itrust.sutd.edu.sg/itrust-labs_datasets/dataset_info/`
Yahoo	`https://webscope.sandbox.yahoo.com/catalog.php?datatype=s&did=70`
KPI	`https://github.com/NetManAIOps/KPI-Anomaly-Detection`

B Jump Diffusion Proof

B.1 Proof Sketch

Under the assumption that the environment process is piecewise constant and predictable, and that the drift, diffusion and the combined jump map satisfy global Lipschitz and linear-growth bounds

Table 6: Comparison of training time and VRAM usage across datasets and models.

Metrics	Model	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
Training time (min)	Sensitive-HUE	16	23	33	72	52	77	91
	IGCL	29	16	32	77	90	205	400
	RedLamp	2	3	1	13	66	5	22
	Ours	15	10	28	59	49	14	99
VRAM (GB)	Sensitive-HUE	0.840	0.952	0.55	0.621	10.533	0.441	0.979
	IGCL	1.233	0.786	1.023	0.823	8.902	0.640	0.823
	RedLamp	0.725	0.380	0.378	0.345	5.212	0.338	0.583
	Ours	0.627	0.533	0.957	0.719	7.217	0.552	0.720

Table 7: Common hyper-parameter settings used in all experiments.

Hyperparameter	Value
λ_{causal}	1
λ_{KL}	0.01
Sliding-window size	200
Training epochs	200

Higham & Kloeden (2005), classical SDE theory guarantees a unique strong solution on each macro-interval. At each jump time, the Lipschitz jump map deterministically updates the state, preserving uniqueness across intervals. For simulation, we partition each interval of length Δt into Euler–Maruyama Kloeden et al. (1995) micro-steps for the diffusion and apply the jump exactly; the only discretization error is $O(\Delta t)$ in mean square, yielding strong convergence of order 1/2.

B.2 Existence and Uniqueness

We assume the environment process E_t is piecewise-constant and predictable ($E_t = E_{t_k}$ for $t \in [t_k, t_{k+1})$ and E_{t_k} is $\mathcal{F}_{t_k^-}$-measurable). Impose global Lipschitz and linear-growth conditions on the diffusion coefficients and the jump map $G(u, e) = u + p_E(u, e)J_E(u, e)$: there exist $L, K > 0$ such that for all $u, v \in \mathbb{R}^d$ and each environment e,

$$\begin{aligned} \|\mu_e(u) - \mu_e(v)\| + \|\sigma_e(u) - \sigma_e(v)\| &\le L\|u - v\|, \\ \|\mu_e(u)\|^2 + \|\sigma_e(u)\|^2 &\le K(1 + \|u\|^2), \\ \|G(u, e) - G(v, e)\| &\le L\|u - v\|, \\ \|G(u, e)\| &\le K(1 + \|u\|). \end{aligned} \tag{1}$$

Spectral normalisation and weight clipping enforce these bounds in practice. Induction over k then yields a unique strong solution: the diffusion part admits a unique solution on (t_k, t_{k+1}), and the Lipschitz jump map deterministically propagates the state to $U_{t_{k+1}}$, preserving uniqueness. Environment-driven variations are captured by μ_E and the soft-jump term $p_E J_E$; any residual deviation therefore signals a causal violation, aligning with our anomaly-detection objective.

B.3 Numerical Approximation and Training

Each macro interval Δt is subdivided into N micro–steps of size $\delta t = \Delta t/N$. For $m = 0, \dots, N-1$ we perform the Euler–Maruyama update

$$\begin{aligned} U_{k,m+1} &= U_{k,m} + \mu_E(U_{k,m})\,\delta t + \sigma_E(U_{k,m})\sqrt{\delta t}\,\epsilon_{k,m}, \\ \epsilon_{k,m} &\sim \mathcal{N}(0, I), \end{aligned} \tag{2}$$

starting with $U_{k,0} = U_{t_k}$. After the N micro–steps we apply the instantaneous soft jump

$$U_{t_{k+1}} = U_{k,N} + p_E(U_{k,N}, E)\, J_E(U_{k,N}, E). \tag{3}$$

Table 8: Dataset-specific hyper-parameter settings. **Input/Latent/Hidden Dim**: dimensionalities of input, latent state, and hidden layers; **K**: number of pre-specified environments; γ: the scalar controlling the strength of jump influence; $\boldsymbol{\lambda}_{\text{causal}}$: weight on causal loss; **Patience**: early-stopping patience; **LR**: learning rate; **WD**: weight decay; **Step**: step size; $\boldsymbol{\gamma}_{\text{sched}}$: decay factor of the learning-rate scheduler.

Dataset	Input Dim	Latent Dim	Hidden Dim	K	γ	$\boldsymbol{\lambda}_{\text{causal}}$	Batch Size	Patience (epochs)	LR	WD	Step (epochs)	$\boldsymbol{\gamma}_{\text{sched}}$
ASD	19	32	64	4	0.8	1	32	15	$5e^{-4}$	$1e^{-5}$	30	0.1
ECG	2	32	64	2	0.7	1	16	10	$1e^{-3}$	$1e^{-4}$	50	0.3
MSL	55	128	256	4	0.7	1.2	32	10	$3e^{-4}$	$1.5e^{-4}$	50	0.4
SMD	38	64	128	4	0.8	1.2	32	10	$5e^{-4}$	$1e^{-4}$	40	0.5
WADI	127	256	512	4	0.8	1	256	8	$1e^{-3}$	$1e^{-4}$	15	0.5
Yahoo	1	32	64	4	0.3	1	16	8	$5e^{-4}$	$1e^{-4}$	15	0.5
KPI	1	32	64	4	0.8	1	64	15	$5e^{-4}$	$1e^{-5}$	30	0.1

Table 9: F1 scores for different sliding-window sizes (higher is better). Best scores are in **bold**

Window	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
50	0.556	0.391	0.358	0.565	0.660	0.707	0.243
100	0.601	0.454	0.412	0.561	0.667	**0.773**	0.279
150	0.632	0.549	0.433	0.564	**0.712**	0.724	0.321
200	0.676	**0.586**	0.467	0.575	0.701	0.703	0.346
250	**0.704**	0.552	**0.495**	**0.589**	0.680	0.679	**0.381**

Because the jump is handled exactly, the only source of discretisation error lies in the diffusion part. Under the Lipschitz and growth conditions above:

$$\max_{k \le K} \mathbb{E}\big[\|U(t_{k+1}) - U_{t_{k+1}}\|^2\big] \le C\,T\,\Delta t, \tag{4}$$

where $T = t_K$ and C depends on L, K but not on k. Thus the scheme converges in mean square with order $1/2$ and provides stable gradients for end-to-end optimisation.

Thus, our piecewise diffusion with soft jump formulation inherits the expressive power of classical jump models, while, thanks to Lipschitz constraints and exact jump handling, retaining both differentiability and solid theoretical guarantees (existence, uniqueness, and numerical convergence).

C Window-size Sensitivity Analysis

Table 9 presents the complete table of CSJD-AD performance across all datasets under varying window sizes (from 50 to 250). We observe that ASD, MSL, SMD, and KPI benefit from longer window sizes (250), while Yahoo, WADI, and ECG achieve better performance with shorter windows (100, 150 or 200). Based on these trends, we select a window size of 200 as a balanced configuration to minimize sensitivity to this hyperparameter across datasets.

D Extended Benchmark Results

The Table 10 is the extended main results including precision and recall scores. The Table 11 and Table 13 is the extended ablation experiment and robustness test results with AUCPR additionally included.

Table 10: Time-series anomaly-detection performance on seven public complete benchmarks with precision, recall, F1, and AUCPPR(higher is better). Best scores are in **bold**; second–best are underlined.

Method	Metric	Multivariate Benchmarks					Univariate Benchmarks	
		ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
LSTM-VAE	Precision	0.245	0.206	0.272	0.268	0.877	0.255	0.135
	Recall	0.521	0.411	0.808	0.580	0.144	0.450	0.271
	F1	0.327	0.274	0.407	0.367	0.248	0.326	0.182
	AUCPR	0.245±.180	0.206±.150	0.285±.249	0.395±.257	0.139	0.255±.152	0.135±.120
OmniAnomaly	Precision	0.167	0.147	0.161	0.306	0.994	0.219	0.133
	Recall	0.414	0.440	0.846	0.912	0.129	0.762	0.411
	F1	0.238	0.216	0.271	0.459	0.229	0.340	0.201
	AUCPR	0.175±.132	0.154±.152	0.149±.182	0.365±.202	0.120	0.245±.218	0.140±.010
AnomalyTran	Precision	0.298	0.325	0.218	0.206	0.057	0.260	0.212
	Recall	0.744	0.812	0.823	0.582	0.434	0.651	0.525
	F1	0.425	0.464	0.344	0.304	0.102	0.372	0.303
	AUCPR	0.281±.201	0.306±.221	0.236±.237	0.273±.232	0.040	0.261±.182	0.204±.139
TranAD	Precision	0.233	0.346	0.290	0.302	0.887	0.392	0.223
	Recall	0.446	0.691	0.759	0.534	0.155	0.630	0.401
	F1	0.305	0.461	0.420	0.386	0.263	0.484	0.287
	AUCPR	0.238±.178	0.368±.251	0.278±.239	0.412±.260	0.139	0.691±.324	0.285±.206
D^3R	Precision	0.150	0.188	0.110	0.237	0.063	0.126	0.094
	Recall	0.751	0.751	0.930	0.526	0.831	0.501	0.375
	F1	0.253	0.301	0.197	0.326	0.117	0.201	0.152
	AUCPR	0.150±.110	0.180±.131	0.138±.101	0.228±.167	0.070	0.120±.080	0.090±.061
PUAD	Precision	0.263	0.285	0.258	0.269	0.955	0.225	0.211
	Recall	0.525	0.570	0.750	0.562	0.150	0.450	0.424
	F1	0.351	0.382	0.384	0.364	0.259	0.301	0.284
	AUCPR	0.280±.203	0.304±.221	0.307±.102	0.291±.210	0.155	0.240±.172	0.224±.152
NPSR	Precision	0.267	0.315	0.240	0.265	0.784	0.413	0.241
	Recall	0.525	0.788	0.839	0.623	0.500	0.825	0.483
	F1	0.350	0.451	0.373	0.372	0.613	0.550	0.321
	AUCPR	0.281±.200	0.405±.281	0.336±.241	0.335±.245	0.552	0.495±.344	0.288±.160
Dual-TF	Precision	0.620	0.480	0.116	0.263	0.504	0.665	0.303
	Recall	0.710	0.610	0.140	0.316	0.605	0.797	0.363
	F1	0.661	0.538	0.127	0.287	0.551	0.725	0.330
	AUCPR	0.628±.212	0.511±.182	0.124±.126	0.215±.074	0.523	0.689±.234	0.314±.107
Sensitive-HUE	Precision	0.286	0.215	0.330	0.295	0.865	0.167	0.099
	Recall	0.505	0.550	0.712	0.608	0.587	0.870	0.602
	F1	0.366	0.309	0.451	0.397	0.699	0.281	0.170
	AUCPR	0.340±.188	0.410±.245	0.432±.121	0.462±.283	0.641	0.489±.429	0.227±.253
IGCL	Precision	0.228	0.366	0.219	0.173	0.447	0.408	0.340
	Recall	0.011	0.054	0.228	0.259	0.007	0.133	0.150
	F1	0.022	0.094	0.223	0.208	0.014	0.201	0.208
	AUCPR	0.079±.066	0.183±.141	0.179±.102	0.126±.132	0.218	0.300±.277	0.206±.198
RedLamp	Precision	0.148	0.110	0.254	0.068	0.756	0.205	0.042
	Recall	0.336	0.334	0.321	0.328	0.532	0.548	0.087
	F1	0.205	0.165	0.284	0.113	0.624	0.299	0.057
	AUCPR	0.154±.103	0.200±.196	0.199±.290	0.128±.140	0.564	0.653±.409	0.089±.129
Ours	Precision	0.686	0.568	0.366	0.538	0.786	0.960	0.269
	Recall	0.666	0.604	0.646	0.617	0.633	0.554	0.486
	F1	**0.676**	**0.586**	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**
	AUCPR	**0.682±.193**	**0.627±.161**	**0.464±.296**	**0.637±.183**	**0.653**	**0.937±.200**	**0.342±.242**

E USE OF LLMS

We used ChatGPT to tidy and standardize LaTeX for mathematical formulas, harmonize notation, perform limited synonym substitutions to keep terminology consistent, and run light grammar checks on select sentences. We did not use LLMs to design the method, analyze data, select or curate results, write the experiments section, or generate synthetic data. We did not provide datasets, labels, or implementation code to the model. All technical content and claims were written and verified by the authors, and every LLM suggestion was reviewed and edited. The paper and results remain fully reproducible from our released code and data.

Table 11: Ablation study of the proposed model. Each column reports F1 scores (higher is better). **Bold** numbers denote the best result for that dataset.

Metrics	Variant	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
F1	w/o U_{F}	0.562	0.573	0.463	0.563	0.682	0.662	0.194
	w/o p_E	0.675	**0.604**	0.445	0.568	0.696	0.698	0.310
	w/o E	0.571	0.552	0.455	**0.575**	0.678	0.618	0.118
	w/o $\mathcal{L}_{\text{causal}}$	**0.682**	0.566	0.464	0.563	0.685	0.703	0.250
	w/o $\mathcal{L}_{\mathrm{KL}}$	0.680	0.578	0.465	0.571	0.681	0.655	0.188
	Default	0.676	0.586	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**
AUCPR	w/o U_{F}	0.559	0.615	0.461	0.621	0.633	0.881	0.210
	w/o p_E	**0.689**	**0.651**	0.449	0.631	**0.668**	0.934	0.309
	w/o E	0.569	0.596	0.454	0.636	0.629	0.821	0.180
	w/o $\mathcal{L}_{\text{causal}}$	0.688	0.612	0.461	0.621	0.636	**0.937**	0.182
	w/o $\mathcal{L}_{\mathrm{KL}}$	0.686	0.620	0.462	0.632	0.632	0.872	0.247
	Default	0.682	0.627	**0.464**	**0.637**	0.653	**0.937**	**0.342**

Table 12: Effect of modifying learned environment settings on F1 and AUCPR performance (higher is better). Best scores for each dataset within a block are shown in **bold**. Default corresponds to training with correctly learned environment variables.

Metrics	Strategy	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
F1	Single E	0.654	0.585	0.463	**0.575**	0.693	**0.703**	**0.306**
	Shuffled E	0.539	0.470	0.404	0.396	0.685	0.501	0.143
	Default	**0.676**	**0.586**	**0.467**	**0.575**	**0.701**	**0.703**	**0.346**
AUCPR	Single E	0.572	0.626	0.450	0.641	0.647	0.936	0.341
	Shuffled E	0.670	0.503	0.409	0.438	0.642	0.537	0.139
	Default	**0.682**	**0.627**	**0.464**	**0.637**	**0.653**	**0.937**	**0.342**

Table 13: Effect of noise and missing-value settings on F1 and AUCPR performance (higher is better). Best scores for each dataset within a block are shown in **bold**. Default corresponds to training with no added noise or missing values.

Metrics	Strategy	Level	ASD	ECG	MSL	SMD	WADI	Yahoo	KPI
F1	Noise level	0.10	0.652	0.584	0.451	0.569	0.697	0.669	0.218
		0.05	0.642	0.594	0.458	0.574	**0.723**	0.694	0.277
		0.01	**0.703**	**0.600**	0.457	0.575	0.715	**0.707**	**0.353**
	Missing ratio	0.40	0.631	0.476	0.454	0.602	0.544	0.702	0.343
		0.20	0.605	0.476	0.455	0.599	0.656	0.702	0.343
		0.10	0.602	0.477	0.453	**0.605**	0.662	0.702	0.343
	Default	—	0.676	0.586	**0.467**	0.575	0.701	0.703	0.346
AUCPR	Noise level	0.10	0.678	0.625	0.451	0.629	0.649	0.859	0.260
		0.05	0.677	0.634	0.456	0.636	**0.676**	0.911	0.295
		0.01	**0.754**	**0.640**	0.456	0.637	0.668	0.932	**0.369**
	Missing ratio	0.40	0.664	0.528	0.453	0.673	0.489	0.935	0.357
		0.20	0.637	0.528	0.454	0.669	0.606	0.935	0.357
		0.10	0.632	0.528	0.452	**0.677**	0.612	0.935	0.357
	Default	—	0.682	0.627	**0.464**	0.637	0.653	**0.937**	0.342